THEORETICAL AND EXPERIMENTAL STUDY OF THE ERROR OF TEMPERATURE MEASUREMENT BY THERMOCOUPLES IN THERMAL-INSULATION MATERIALS

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On the basis of a numerical study of the solution of a three-dimensional unsteady heat-conduction problem, we formulate practical recommendations for the choice of thermocouples to measure temperatures in thermal-insulation materials.

The contact method of measuring temperatures with thermocouples has been widely used in the experimental study of temperature distributions in solids [1-2]. However, together with its obvious advantages, this method has an inherent shortcoming: the thermocouple indicates the temperature of the junction itself, which is generally different from the actual temperature at the point of measurement. This difference causes a systematic error in temperature measurements made by the contact method [2].

It was shown in [3, 4] that when a thermocouple is located along the normal to the heated surface of a sample of thermal-insulation material, the error may reach 50-70% of the actual temperature. The distortion can be substantially reduced if the thermoelectrodes of the thermocouple are placed parallel to the heated surface of the sample, i.e., in an isothermal plane [5]. At the present only approximate methods of estimating the accuracy of such measurements are known [1].

We consider a two-electrode thermocouple with a junction whose thermoelectrodes are located in an isothermal plane of the sample under study. To simplify the discussion we replace the actual physical model of the thermocouple and the sample under study by the following computational scheme.

In the region of the sample under study where the thermocouple is located we select a rectangular parallelepiped Ω_1 with edges of length L_1, L_2, and L_3 (Fig. 1) such that the distortion of the temperature distribution on its faces resulting from the presence of the thermocouple is negligibly small. One face of the parallelepiped is heated by a radiation flux $q_{in}(\tau)$ which is uniform over the surface but varies with time. This surface is cooled simultaneously by natural convection (heat-transfer coefficient α) and by self-radiation. The opposite face and the lateral surfaces of the parallelepiped are thermally insulated. The thermocouple is placed in the sample under study at a depth l.

We replace the actual cross section S_T of the thermoelectrodes Ω_2 and Ω_3 (a circle of diameter d_T) by a square cross section S_r of side $r = d_T\sqrt{\pi}/2$ which has the same area as the actual circular cross section. We assume that the temperature of the free ends of the thermoelectrodes is constant and equal to the ambient temperature T_{am} and that there is perfect thermal contact between the material Ω_1 under study and the thermoelectrodes Ω_2 and Ω_3 of the thermocouple.

Under the assumption that there are no physicochemical transformations in the system of bodies under study, we formulate the following mathematical model of the problem posed:

$$\rho_m C_m(T)\frac{\partial T_m}{\partial \tau} = \frac{\partial}{\partial x}\left[\lambda_{xm}(T)\frac{\partial T_m}{\partial x}\right] + \frac{\partial}{\partial y}\left[\lambda_{ym}(T)\frac{\partial T_m}{\partial y}\right] + \frac{\partial}{\partial z}\left[\lambda_{zm}(T)\frac{\partial T_m}{\partial z}\right], \quad (1)$$

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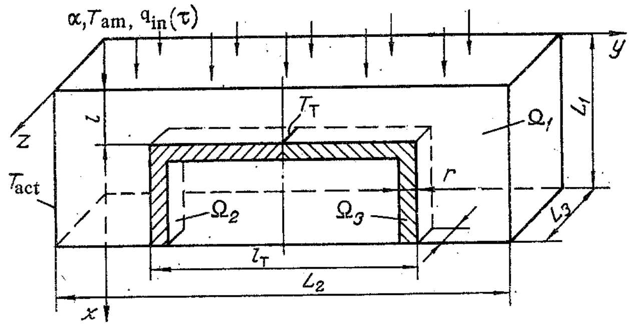

Fig. 1. Computational scheme.

in the domain $(x,\ y,\ z) \in \Omega_m\ (m = 1,\ 2,\ 3)$:

$$\tau = 0, \quad T_m(x,\ y,\ z,\ 0) = T_0; \tag{2}$$

$$x = 0, \ \lambda_{x1} \frac{\partial T_1}{\partial x} = \alpha (T_1 - T_{am}) + \varepsilon_1 (T) [\sigma T_1^4 - q_{in}\ (\tau)]; \tag{3}$$

$$x = l, \ \lambda_{x1} \frac{\partial T_1}{\partial x} = \begin{cases} \lambda_{x2} \dfrac{\partial T_2}{\partial x} & \text{for } (x = l,\ y,\ z) \in \Omega_2; \\ \lambda_{x3} \dfrac{\partial T_3}{\partial x} & \text{for } (x = l,\ y,\ z) \in \Omega_3; \end{cases} \tag{4}$$

$$x = L_1, \ \frac{\partial T_1}{\partial x} = 0 \quad \text{for } (x = L_1,\ y,\ z) \in \Omega_1;$$

$$T_2 = T_{am} \quad \text{for } (x = L_1,\ y,\ z) \in \Omega_2, \quad T_3 = T_{am} \quad \text{for } (x = L_1,\ y,\ z) \in \Omega_3; \tag{5}$$

$$\lambda_{s1} \frac{\partial T_1}{\partial S}\bigg|_{\Gamma_1} = \lambda_{s2} \frac{\partial T_2}{\partial S}\bigg|_{\Gamma_1}, \quad T_1|_{\Gamma_1} = T_2|_{\Gamma_1}; \tag{6}$$

$$\lambda_{s1} \frac{\partial T_1}{\partial S}\bigg|_{\Gamma_2} = \lambda_{s3} \frac{\partial T_3}{\partial S}\bigg|_{\Gamma_2}, \quad T_1|_{\Gamma_2} = T_3|_{\Gamma_2}; \tag{7}$$

$$\lambda_{s2} \frac{\partial T_2}{\partial S}\bigg|_{\Gamma_3} = \lambda_{s3} \frac{\partial T_3}{\partial S}\bigg|_{\Gamma_3}, \quad T_2|_{\Gamma_3} = T_3|_{\Gamma_3}; \tag{8}$$

$$\frac{\partial T_m}{\partial S}\bigg|_{\Gamma_4} = 0, \quad m = 1,\ 2,\ 3, \tag{9}$$

where Γ_1 and Γ_2 are the surfaces of contact of the lateral surfaces of the thermoelectrodes and the material, Γ_3 is the surface of contact (junction) of the thermoelectrodes, Γ_4 is the lateral surface of the parallelepiped, and S is the normal to the surface under consideration.

Problem (1)-(9) is a nonlinear nonstationary boundary value problem for a domain with discontinuous coefficients. We solved it by the total approximation method [6]. We replaced the three-dimensional heat-conduction equation (1) by a chain of one-dimensional equations which we approximated by a two-layer difference scheme on a three-point pattern [6]. We approximated the boundary conditions in conformity with the energy balance method. We solved each difference equation by the pivotal method.

The numerical solution determined the three-dimensional unsteady temperature distribution in the system of bodies considered. A FORTRAN-IV program was written to carry out the algorithm developed.

An experimental test of the mathematical model formulated and the method of solving it consisted in the following.

Three samples of kaolin fiber 30 × 30 × 30 mm were prepared. The surface of the samples to be heated was covered with a black high-melting coating based on borosilicate glass which

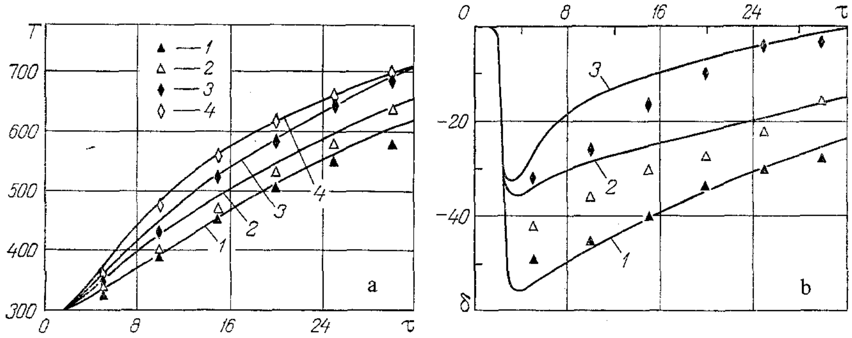

Fig. 2. a) Values of the temperatures recorded by thermocouples; b) variation of relative error. The curves were calculated and the points are experimental: 1) l_T/d_T = 10; 2) 30; 3) 100 (Chromel–Alumel 0.2 mm in diameter; 4) T_{act}, l_T/d_T = 300 (Chromel–Alumel 0.05 mm in diameter). T in °K, τ in sec, δ in %.

was opaque to radiation. Three Chromel–Alumel thermocouples 0.2 mm in diameter and one 0.05 mm in diameter were placed at the same depth l = 3 mm in each sample. The ratio of the length of the thermoelectrodes positioned along an isotherm to the diameter of the thermocouple was l_T/d_T = 10, 30, and 100 for the 0.2-mm-diameter thermocouple, and l_T/d_T = 300 for the 0.05-mm-diameter thermocouple. The thermocouple readings were recorded by a KSP-4 automatic potentiometer.

The samples were heated on a radiation heating bench [7]. The radiation flux density incident on the surface of a sample recorded by an asymptotic sensor of the Gardon type [2] was $q_{in}(\tau)$ = 120 kW/m². The ambient temperature was $T_{am} = T_0$ = 293°K. In accordance with the dimensionless relations in [8], the heat-transfer coefficient for natural convection at the heated surface of a sample was taken equal to α = 12 W/m²·°K. The thermal insulation of the lateral surfaces of the samples ensured one-dimensional heating of the material under study.

Since the largest pores of the kaolin fiber (0.001-0.004 mm) are appreciably smaller than the diameter of a thermocouple junction (0.3-0.5 mm), thermal contact in their region of contact can be considered close to perfect [9].

The values of the thermophysical properties of the material under study used in the calculations were taken from [9], and those of the thermoelectrodes of the Chromel–Alumel thermocouples from [10]. The emissivity of the opaque coating based on borosilicate glass was taken equal to 0.9.

In the calculations we considered a L_1 = 30, L_2 = 30, L_3 = 2.5 mm region near the thermocouple. We determined the space–time net of the approximate discrete model of problem (1)-(9) by comparing numerical and analytic solutions of a linear nonstationary heat-transfer problem. The latter was obtained by the method of finite integral transforms [11]. We assumed that the properties of the material under investigation in the regions Ω_1, Ω_2, and Ω_3 are constant and independent of temperature, and that there is no self-radiation from the surface of the sample.

A comparison of the two solutions showed that for the finite difference approximation of problem (1)-(9) with a 0.2-sec time step and a nonuniform space net with 35 × 35 × 10 nodes for L_1, L_2, and L_3, respectively, the error of the numerical solution obtained for the temperature does not exceed 3% of the analytic solution.

The calculated values of the temperature T_T of the thermocouple junction and the experimental thermocouple readings are shown in Fig. 2a. The experimental values of the temperatures are the arithmetic mean of three measured values. The accuracy of the determination of the average result was found with a confidence coefficient of 0.95. The maximum mean-square error of the results over the whole range of measurements did not exceed 9.7%.

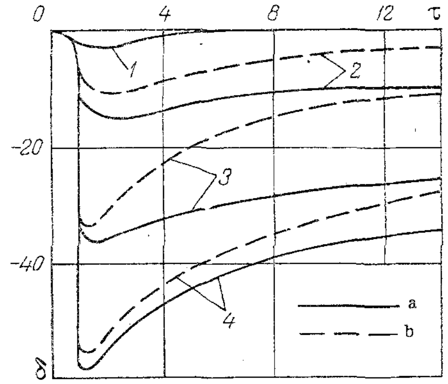

Fig. 3. Error for thermoelectrodes of various diameters. 1) 0.05; 2) 0.1; 3) 0.2; 4) 0.5 mm. a) $l_T/d_T = 30$; b) $l_T/d_T = 100$.

Figure 2a shows that the readings of the 0.05-mm diameter thermocouple are in satisfactory agreement with the calculated values of the actual temperature T_{act}. The values of the temperatures recorded by the 0.2-mm-diameter thermocouples agree with the calculated values of T_T. The maximum difference is 25°K, which lies within the limits of experimental error.

The relative error was defined as

$$\delta\% = \frac{T_T - T_{act}}{T_{act} - T_{am}} 100\ \%,$$

where T_{act} is the calculated temperature of the sample outside the range of the perturbing action of a thermocouple at depth $x = l$. In the experimental determination of the error, the reading of the 0.05-mm-diameter thermocouple was taken as the actual temperature. Figure 2b shows the variation of the relative error of the temperature measurement by the 0.2-mm-diameter thermocouples. It is clear that at early times the error may reach 30-50%, and the absolute difference between the actual and distorted temperatures is 70-100°K (Fig. 2). For the relatively large value $l_T/d_T = 100$ this difference reaches 20-25°K. The substantial increase in the relative error in certain time intervals can be accounted for by the large temperature gradients which develop along the thermoelectrodes of a thermocouple at the instant the front of the temperature wave reaches the thermocouple junction. As the material is heated up, the temperature profile in the sample under study is flattened, and the error decreases monotonically (Fig. 2).

The satisfactory agreement of the calculated and experimental values of the temperatures and the magnitudes of the relative error show that the mathematical model of the problem corresponds to the actual thermal processes occurring in the sample under study and in the thermocouple.

Let us consider the question of the choice of thermocouples for the correct measurement of temperatures in thermal-insulation materials with thermal conductivities of the order of 0.1 W/m·°K. The input data for the calculations are the same as in the preceding case.

The results of the investigation are shown in Fig. 3. In using thermocouples having diameters less than 0.05 mm to measure temperatures in thermal-insulation materials, the relative error does not exceed 3%. Thermoelectrodes located on an isothermal surface should be more than 1.5 mm long. The use of thermocouples with diameters greater than 0.1 mm to measure temperatures in thermal-insulation materials ($\lambda \sim 0.1$ W/m·°K) leads to errors of 10-30% even for relatively long thermoelectrodes (Fig. 3).

Investigations performed within the framework of the present study showed that the recommendations formulated are valid for tungsten—tungsten-rhenium, platinum—platinum-rhodium, Chromel—alumel, and Chromel—Copel thermocouples for various rates of heating of the material under study.

Similar calculations were performed for thermal-insulation materials of the fiberglass-reinforced plastic type with thermal conductivities on the order of 0.5-1 W/m·°K. The results obtained agree with the recommendations on the choice of thermocouples to measure temperatures in fiberglass-reinforced plastics made in [5].

NOTATION

T_T, temperature of region under study; T_0, initial temperature of system of bodies considered; τ, time; ρ_m, density; C_m, specific heat; λ_{xm}, λ_{ym}, λ_{zm}, components of thermal conductivity tensor; ε_1, emmissivity of material under study; σ, Stefan–Boltzmann constant.

LITERATURE CITED

1. N. A. Yaryshev, Theoretical Bases of the Measurement of Unsteady Temperatures [in Russian], Énergiya, Leningrad (1967).
2. A. N. Gordov, Ya. V. Malkov, N. N. Érgardt, and N. A. Yaryshev, Accuracy of Contact Methods of Temperature Measurement [in Russian], Standartov, Moscow (1976).
3. B. Bek, "Distorting effect of a thermocouple on the temperature distribution in materials with a low thermal conductivity," Teploperedacha, No. 2, 33-42 (1962).
4. V. N. Eliseev, V. I. Vorotnikov, V. A. Solovov, and V. A. Tovstonog, "Investigation of systematic error of temperature measurement in semitransparent materials with a low thermal conductivity," Izv. Vyssh. Uchebn. Zaved., Mashinostr., 1, 157-159 (1982).
5. Yu. V. Polezhaev and F. B. Yurevich, The Thermal Shield [in Russian], Énergiya, Moscow (1976).
6. A. A. Samarskii, Theory of Difference Schemes [in Russian], Nauka, Moscow (1977).
7. G. B. Sinyarev, "Bench for the study of the thermally stressed state of structures," Catalog: Moscow Higher Technical School Machines, Devices, and Benches, Moscow (1974).
8. M. A. Mikheev and I. M. Mikheeva, Fundamentals of Heat Transfer [in Russian], Énergiya, Moscow (1977).
9. R. E. Krzhizhanovskii and I. I. Chudnovskaya, "Investigation of the thermal insulating properties of kaolin fibers," Teplofiz. Vys. Temp., 4, 355-359 (1966).
10. O. A. Gerashchenko and V. G. Fedorov, Heat and Temperature Measurements (Reference Handbook) [in Russian], Naukova Dumka, Kiev (1965).
11. V. N. Eliseev, "Generalized solution of a three-dimensional unsteady heat-conduction problem for bodies with the volume absorption of radiant energy," Proc. Moscow Higher Technical School, No. 205 [in Russian], Moscow (1974), pp. 5-43.